

Forests, groves and Higgs bosons

Gauge invariance classes in spontaneously broken gauge theories

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Received: 2 June 2003 / Revised version: 21 July 2003 /

Published online: 5 September 2003 – © Springer-Verlag / Società Italiana di Fisica 2003

Abstract. We determine the gauge invariance classes of tree level Feynman diagrams in spontaneously broken gauge theories, providing a proof for the formalism of gauge and flavor flips. We find new gauge invariance classes in theories with a nonlinearly realized scalar sector. In unitarity gauge, the same gauge invariance classes correspond to a decomposition of the scattering amplitude into pieces that satisfy the relevant Ward identities individually. In theories with a linearly realized scalar sector in R_ξ gauge, no additional non-trivial gauge invariance classes exist compared to the unbroken case.

1 Introduction

The agreement of theoretical predictions derived from the standard model (SM) of electroweak interactions and experiment has been established to an impressive degree. The only missing ingredient is the Higgs boson that has yet to be discovered. The electroweak SM can nevertheless only be a low energy approximation to a more fundamental theory that should become visible at TeV scale energies. Indications on the nature of the underlying theory should be found by experiments at the LHC or a future linear collider (see e.g. [1]). Channels with many tagged particles open up at these experiments and challenge theorists to make precise predictions for processes with many particles in the final state.

Assuming a Higgs boson will be found in future experiments, determining its quantum numbers and couplings will require the study of processes with many fermions in the final state [1]. Examples are the measurement of the triple Higgs self-coupling [2] and of the top–Higgs Yukawa coupling [3] via associated top–Higgs production. In the latter example, there are five tree level diagrams contributing to the signal process $e^+e^- \rightarrow t\bar{t}H$, while almost forty-thousand diagrams contribute to the corresponding observable eight fermion final states like $e^+e^- \rightarrow b\mu^+\nu_\mu\bar{b}d\bar{u}b\bar{b}$. Existing calculations of such irreducible backgrounds [4] classify the contributing diagrams according to their topology in order to perform the phase space integration, arriving at a gauge invariant result only after the resulting integrals are added.

To disentangle signal and background diagrams in a gauge invariant way, it is desirable to find separately gauge invariant subsets of Feynman diagrams (FDs), so called “gauge invariance classes” (GICs). After a classification of GICs in four fermion production processes [5], a systematic procedure to construct minimal GICs or “groves” of tree level diagrams using the formalism of “gauge and flavor flips” has been found in [6]. However, a detailed discussion of GICs involving Higgs bosons has not yet been given.

In this work, we will clarify the role of Higgs bosons in the GICs. We provide a proof of the formalism of [6] for spontaneously broken gauge theories (SBGTs), based on a diagrammatic analysis of the Slavnov–Taylor identities (STIs). We will find that the brief discussion in [6] is justified for a linear realization of the symmetries of the SM and no new non-trivial groves arise compared to an unbroken gauge theory. However, additional groves appear in the case of non-linearly realized symmetries [7]. These groves are also consistent in unitarity gauge, i.e. the corresponding amplitudes satisfy the Ward identities (WIs).

As an example, consider Higgs production via the two diagrams shown in Fig. 1. According to our results, the Higgsstrahlung and the vector boson fusion diagram be-

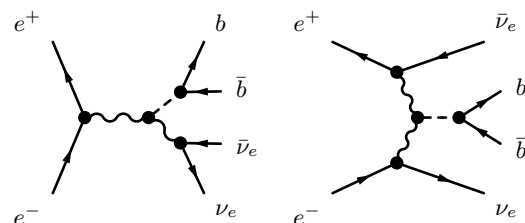


Fig. 1. Higgsstrahlung and vector boson fusion

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long to different GICs, both in the linear and the non-linear representation. In a non-linear representation, however, both diagrams are gauge invariant *by themselves* (provided the electrons are taken as massless) while in the linear representation they are part of larger GICs including gauge boson exchange diagrams.

In Sect. 2 we review GICs in unbroken gauge theories, sketch the formalism of gauge and flavor flips and present the correct flips for SGBTs. A summary of our graphical notation for STIs is given in Sect. 3 before we present the diagrammatic derivation of GICs in Sect. 4. The correct definition of the gauge flips in SGBTs is discussed in Sect. 5, both for the linear and the non-linear realization. The structure of the GICs in SGBTs is analyzed in Sect. 6.

2 Gauge invariance classes and flips

Before we turn to a formal derivation of the formalism from the underlying STIs, we will briefly review existing results for GICs in unbroken gauge theories and give the correct form of the gauge flips in SGBTs.

Physical scattering amplitudes in gauge theories satisfy the simple WI

$$-ik_\mu \mathcal{M}^\mu(\text{in} + A \rightarrow \text{out}) = 0 \quad (1a)$$

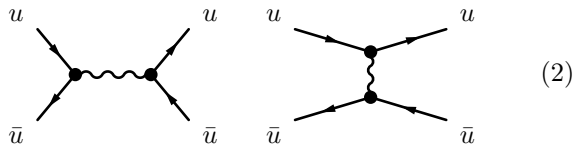
in unbroken gauge theories and

$$-ik_\mu \mathcal{M}^\mu(\text{in} + W \rightarrow \text{out}) = m_w \mathcal{M}(\text{in} + \phi \rightarrow \text{out}) \quad (1b)$$

in SGBTs. For the application to tree level diagrams, it is a sufficient requirement to define GICs as (minimal) subsets of FDs that satisfy the WIs and are independent of the gauge fixing parameters. Keeping in mind a future application to loop diagrams, we will go further in our proof presented in Sect. 4 and demand that the GICs satisfy the appropriate STIs when some of the external particles are off their mass shell. However, the simpler definition in terms of WIs suffices for the present introductory discussion.

2.1 Gauge invariance classes in unbroken gauge theories

As a trivial first example, consider the process $u\bar{u} \rightarrow u\bar{u}$ in QCD. Here a s - and a t -channel diagram contribute:



Both diagrams are separately gauge invariant (i.e. are independent of the gauge parameter in the gluon propagator), as can be seen without calculation from the following observation: in the process $u\bar{u} \rightarrow c\bar{c}$ where both fermion pairs belong to different families, only the s -channel diagram appears, while in the case of $u\bar{c} \rightarrow u\bar{c}$ only the t -channel diagram appears. Since the scattering amplitudes

for physical processes are gauge invariant, both diagrams in (2) must also be gauge invariant by themselves.

This is a simple example of a “flavor selection rule”. The separate gauge invariance can of course be verified easily by an explicit calculation, but it was shown in [6] how the flavor selection rule argument carries over to more complicated situations.

The argument leading to the flavor selection rules does not depend on the existence of different flavors of quarks in the SM, since one can always introduce fictitious additional generations, leading to a conserved quantum number that has to be conserved along quark lines passing through the diagrams. This allows one to extend the formalism to diagrams with an arbitrary number of external fermions.

As an application to a five point function, consider the amplitude for the process $\bar{q}q \rightarrow \bar{q}qg$. Because of the flavor selection rules, it contains two GICs, resulting from the insertion of the gluon into the s - and t -channel diagrams for the process $\bar{q}q \rightarrow \bar{q}q$. The GIC obtained by inserting the gluon in the s -channel diagram is

(3)

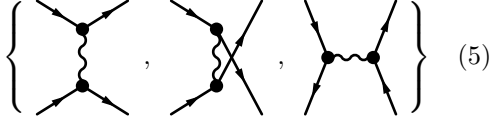
The situation simplifies further in QED, because there is no triple photon vertex. Indeed, it is well known in QED that the expression obtained from a given FD by summing over all possible insertions of a photon along a charge carrying fermion line going through the diagram, satisfies the WI by itself. Thus the amplitude for $e^+e^- \rightarrow \mu^+\mu^-\gamma$ can be separated further into two gauge invariant subsets:

(4)

This allows us to have a separate treatment of initial-state and final-state Bremsstrahlung. If we consider instead the process $e^+e^- \rightarrow e^+e^-\gamma$, i.e. Bhabha scattering with an additional Bremsstrahlung-photon, we can appeal to the flavor selection rules discussed above and see that we get altogether four GICs.

In order to construct minimal GICs of tree diagrams in non-abelian gauge theories systematically, the formalism of “gauge and flavor flips” was introduced in [6]. An elementary “flavor flip” is defined as an exchange of two diagrams in the set

$$F_4 = \{F_4^i | i = 1, 2, 3\} =$$

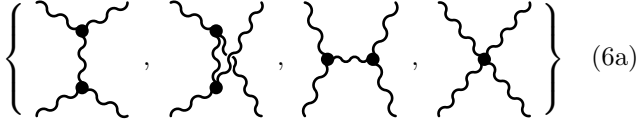
 (5)

that is compatible with the Feynman rules.

In the example of the four point function, the two diagrams in (2) are connected by a flavor flip $F_4^1 \leftrightarrow F_4^3$. Flips between pairs of larger diagrams are obtained by applying elementary flips to four particle subdiagrams [6].

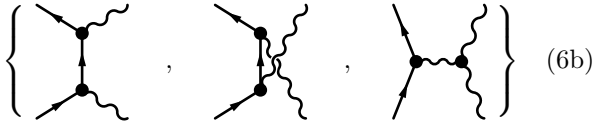
Elementary “gauge flips” are defined as exchanges of diagrams in the sets

$$G_4 = \{G_4^i | i = 1, 2, 3, 4\} =$$

 (6a)

and

$$G_{4,2F} = \{G_{4,2F}^i | i = 1, 2, 3\} =$$

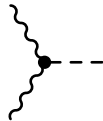
 (6b)

respectively. For instance, the diagrams in the GIC (3) are connected by gauge flips of subdiagrams from $G_{4,2F}$.

A set of diagrams connected by flavor and gauge flips is called a “forest”, while a set of diagrams connected by gauge flips only is called a “grove”. The example of the $\bar{q}q \rightarrow \bar{q}qg$ amplitude suggests that the groves can be identified with GICs. Indeed it has been shown in [6], that the groves are the *minimal* GICs of FDs. Furthermore, the forest of FDs is connected and consists of all FDs contributing to the amplitude. Therefore the formalism can be used to implement a FD generator [8] that generates the groves en passant. Further results and examples for the structure of groves in the electroweak SM can be found in [6, 9, 10].

2.2 Flips in spontaneously broken gauge theories

In a SGBT, the role of Higgs bosons in gauge and flavor flips is not clear a priori. Because of the presence of a WWH vertex (where W denotes an arbitrary massive gauge boson)



neutral Higgs bosons cannot be assigned a (fictitious) conserved quantum number as we have done above to derive the flavor selection rules. Thus it seems plausible that no

new groves should appear compared with unbroken gauge theories. This corresponds to the ad hoc prescription given in [6] for the construction of the gauge flips, that proposes to treat Higgs bosons like gauge bosons.

We show in Sect. 5 that this intuitive argument is essentially correct, provided a linear representation of the scalar sector is used. A richer structure of the groves will emerge in theories with a non-linearly realized symmetry,

The appearance of new groves in non-linear realizations is very plausible if one considers the non-linearly realized electroweak SM [11]. Here the Higgs boson transforms trivially under gauge transformations and can be removed from the theory without spoiling gauge invariance. The trivial transformation law implies that the diagrams without Higgs bosons form GICs by themselves, in contrast to the linear parameterization. Therefore one can simplify the elementary gauge flips by omitting the internal Higgs bosons. The case of a more general Higgs sector with charged Higgs bosons requires more careful considerations and is discussed in Sect. 5.2.

Instead of (6b), the flips for $\bar{f}f \rightarrow WW$ are in a linear representation of the symmetry:

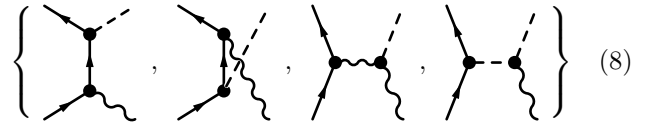
$$\tilde{G}_{4,2F} = G_{4,2F} \cup \{\tilde{G}_{4,2F}^4\} = \{\tilde{G}_{4,2F}^i | i = 1, 2, 3, 4\} =$$

 (7)

As discussed above, the Higgs exchange diagram $\tilde{G}_{4,2F}^4$ is not present for non-linear symmetries. Similarly in the linear representation, the Higgs exchange diagrams have to be included in the gauge flips for the four gauge boson amplitude, cf. (76a), while they do not contribute in the non-linear representation.

As we will discuss in Sect. 5.2, the simplifications in non-linear representations affect only the WWH vertex, while the STI for the WHH vertex is similar in both realizations. Thus diagrams with internal Higgs bosons cannot be left out of the gauge flips if a WHH vertex appears. An example is provided by the gauge flips for $\bar{f}f \rightarrow WH$. Here the internal Higgs boson has to be included both in the linear and non-linear parameterization:

$$\tilde{G}_{4,1H2F} = \{\tilde{G}_{4,1H2F}^i | i = 1, 2, 3, 4\} =$$

 (8)

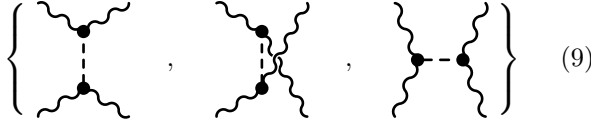
The complete set of gauge and flavor flips is given in Appendix B, both for linear and non-linear representations.

To define the complete forest, additional types of flips have to be introduced compared to the unbroken case. Four particle diagrams with only external Higgs bosons and matter fields are found to be gauge parameter independent by themselves, so we have to introduce another class of flips, that plays a role similar to the flavor flips

and will be called “Higgs flips”. They consist of the four diagrams $\tilde{H}_{4,2F}$ contributing to the $\bar{f}f \rightarrow HH$ amplitude and the seven diagrams \tilde{H}_4 contributing to the four Higgs amplitude. They are given in (79) in the appendix.

In a linear representation, the forest for a given set of external diagrams is defined as the set of diagrams connected by flavor, Higgs and gauge flips while the definition of the groves remains as before.

In non-linear realizations, one has to introduce yet another class of flips that generate the diagrams not needed for the gauge flips, i.e. the exchange $\tilde{G}_{4,2F}^3 \leftrightarrow \tilde{G}_{4,2F}^4$ from (7) for the two fermion two gauge boson function and flips from the diagrams of (6a) to



for the $4W$ function. These “Higgs exchange flips” have to be included in the definition of the forest, if it is to remain connected.

The structure of the groves in SGBTs is analyzed in Sect. 6. Readers who are primarily interested in applications of the formalism can jump to this section directly. It can be read independently from the derivation of the results in the more formal Sects. 3 to 5.

3 Graphical notation for STIs

In the diagrammatic derivation of the GICs in Sect. 4, we will use the STIs for irreducible vertices as building blocks. As a preparation, we need to set up a notation for the STIs for the irreducible vertices. Our notation for the BRS transformations and the STIs for Green’s functions (GFs) is reviewed in Appendix A.

The symmetry of the effective action leads to the Zinn-Justin equation [12] that implies the STIs for the irreducible vertices. To derive the Zinn-Justin equation, one adds sources Ψ^* for the BRS transforms of the fields Ψ to the effective action Γ_0 :

$$\Gamma = \Gamma_0 + \sum_{\Psi} \int d^4x \operatorname{tr}[\Psi^*(\delta_{\text{BRS}}\Psi)], \quad (10)$$

from which the one particle irreducible vertices are obtained by taking functional derivatives with respect to the classical fields

$$\begin{aligned} \frac{i\delta^n \Gamma(\Phi_{\text{cl}})}{\delta\Phi_{\text{cl}}(x_1) \dots \delta\Phi_{\text{cl}}(x_n)} &\equiv \Gamma_{\Phi_1 \dots \Phi_n}(x_1 \dots x_n) \\ &= \langle \Phi(x_1) \dots \Phi(x_n) \rangle^{\text{1PI}}. \end{aligned} \quad (11)$$

The BRS invariance of the effective action in anomaly free theories implies the Zinn-Justin equation

$$\sum_{\Psi} \int d^4x \frac{\delta_L \Gamma}{\delta \Psi^*} \frac{\delta_R \Gamma}{\delta \Psi} + B \frac{\delta_R \Gamma}{\delta \bar{c}} = 0. \quad (12)$$

To derive the STI for the three point vertex, we take the derivative of (12) with respect to two fields Φ and one ghost field and set the classical fields and sources to zero:

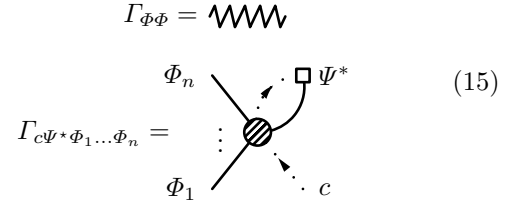
$$\begin{aligned} 0 &= \sum_{\Psi} \int d^4x \{ \Gamma_{c_a \Psi^*} \Gamma_{\Psi \Phi_1 \Phi_2} \\ &\quad + \left[\Gamma_{c_a \Psi^* \Phi_1} \Gamma_{\Psi \Phi_2} + \frac{\delta B(x)}{\delta \Phi_1} \Gamma_{c_a \bar{c} \Phi_2} + (1 \leftrightarrow 2) \right] \}. \end{aligned} \quad (13)$$

Repeating this procedure for an additional derivative, we can derive the corresponding relation for the irreducible four point function, etc.

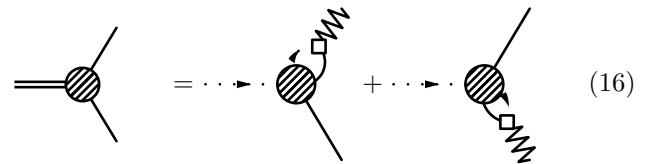
To see the physical content of these identities, we note that the vertex $\Gamma_{c_a \Psi^*}$ is only present for gauge bosons and Goldstone bosons (GBs). Since these contributions are linear in the fields, they do not require radiative corrections beyond the renormalization of the Lagrangian and we get to all orders in perturbation theory

$$\begin{aligned} \text{F.T.} \sum_{\Psi} \int d^4x \Gamma_{c_a \Psi^*} \Gamma_{\Psi \dots} &= -ip_{\mu} \Gamma_{W_a \dots}^{\mu} - m_{W_a} \Gamma_{\phi_a \dots} \\ &\equiv \langle \mathcal{D}_a(p) \dots \rangle^{\text{1PI}}, \end{aligned} \quad (14)$$

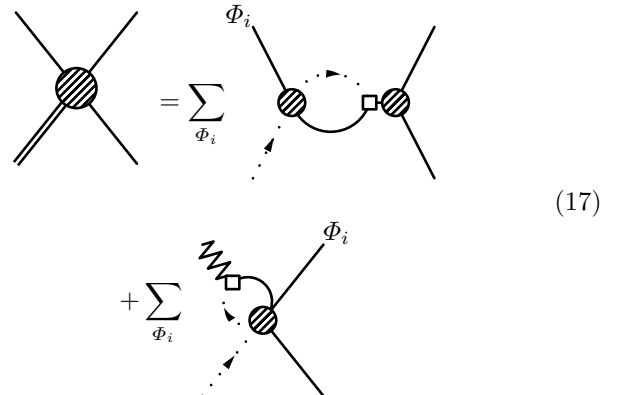
where we have introduced the shorthand \mathcal{D} for the combination of the scalar gauge boson component and the GB. To illustrate the STIs for the irreducible vertices, we introduce the graphical notation



The STI (13) for the three point function reads in this notation



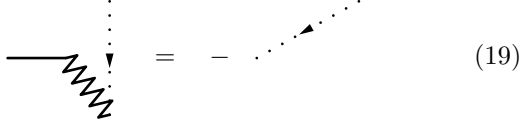
where we have not displayed the term $\propto \delta B/\delta \Phi$ that will be discussed below. The STI for the four point function is written as



The graphical representation of the fact that the irreducible two point function is the inverse of the propagator

$$\int d^4y \Gamma_{\Phi\Phi}(x, y) D_{\Phi\Phi}(y, z) = -\delta^4(x, z) \quad (18)$$

is given by



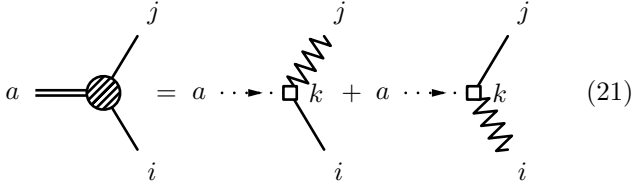
$$(19)$$

This relation will be essential in relating the STIs for GFs and the STIs for irreducible vertices in the diagrammatical derivation of the GICs in Sect. 4.

On tree level in a linear realization of the symmetry, the vertex functions with the insertions of sources for the BRS transformed fields can be read from the BRS transformation. The STI for the three point function (13) becomes

$$\begin{aligned} & -\langle \mathcal{D}_a(p) \Phi_i(k_i) \Phi_j(k_j) \rangle^{\text{1PI}} \\ & = T_{ki}^a \langle \Phi_k(p + k_i) \Phi_j(k_j) \rangle^{\text{1PI}} + T_{kj}^a \langle \Phi_i(k_i) \Phi_k(p + k_j) \rangle^{\text{1PI}} \end{aligned} \quad (20)$$

In the graphical notation of (15) this is written as



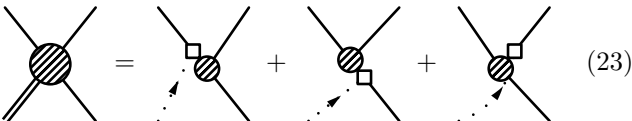
$$(21)$$

For vertices involving gauge bosons or GBs we get additional contributions from the term $\propto \delta B / \delta \Phi$ in the Zinn-Justin equation. Using the equation of motion of the Nakanishi–Lautrup field B , we find that for every gauge boson or GB that is not contracted, there appears a term in the STI for three point functions

$$i \frac{1}{\xi} p_b^\nu \langle c_a(p_a) \bar{c}_b(p_b) \Phi_i(k_i) \rangle^{\text{1PI}} = \frac{1}{\xi} \dots \text{diagram} \quad (22a)$$

$$m_{W_b} \langle c_a(p_a) \bar{c}_b(p_b) \Phi_i(k_i) \rangle^{\text{1PI}} = \dots \text{diagram} \quad (22b)$$

On tree level in a linear representation of the symmetry, the terms with more than one derivative acting on the BRS transforms vanish so the graphical representation of the STI for the four point functions (17) simplifies to

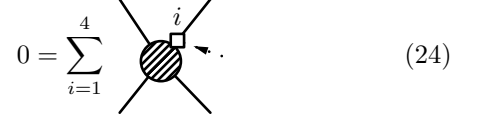


$$(23)$$

Here and in (24) below, additional terms from the derivatives of the BRS transforms appear for non-linearly realized symmetries and will be discussed in subsection 5.2.

In higher orders of perturbation theory and for non linearly realized symmetries, additional contributions from the term involving the Nakanishi–Lautrup field B appear for external gauge and Goldstone bosons, similar to (22).

In a renormalizable theory, there is no five point vertex on tree level, so taking four derivatives of (12) with respect to physical fields we get



$$(24)$$

We will also need STIs in non-renormalizable theories with non-linearly realized symmetries. Since tree level calculations in effective field theories correspond by Weinberg's power counting theorem to the lowest order in the energy expansion, we do not consider additional non-renormalizable operators. This will no longer suffice for the extension of the formalism to loop diagrams. Also, higher dimensional operators involving Goldstone bosons have to be included to maintain gauge invariance in the non-linear realization.

4 Graphical derivation of gauge invariance classes

4.1 Definition of gauge invariance classes

On tree level, the definition of GICs in terms of the WIs given in Sect. 2 is sufficient. However, for future applications in loop calculations, the off-shell structure of the groves has to be clarified as well. The natural extension of the definition of Sect. 2 is to demand that the GICs satisfy the appropriate STIs (see Appendix A) instead of the WIs. To give a definition, we first have to clarify the notion of a subset of diagrams satisfying a STI, including the definition of the set of contact terms in the STI associated to the subset.

In this work, we will always consider GFs where the propagators of the external particles are not amputated, because the STIs are more familiar in this case. Identities for off-shell amplitudes with amputated external particles are more suitable for numerical calculations and have been considered in detail in [13]. The amputation procedure adds notational complexity, but the conclusions remain unchanged.

To define the form of the contact terms, we introduce a mapping \mathcal{F} that maps every FD to the corresponding contact terms. Note that this is a purely formal mapping and in general it is not true that a contraction of a gauge boson in the original diagram results in the contact terms generated by this mapping. In the following, gray blobs denote a subdiagram, while white blobs denote subamplitudes or subgroves, i.e. sets of diagrams.

Definition 1 The action of \mathcal{F} on a diagram with the insertion of a gauge boson into an external line is given by

$$(25a)$$

The action of \mathcal{F} on diagrams with an insertion of a gauge boson into an internal gauge boson line is defined as follows: replace internal gauge bosons by ghosts in all possible ways until the external particles are reached. In general, one original diagram can correspond to more than one contact term, e.g.

$$(25b)$$

For each external gauge boson or GB, the inhomogeneous terms in the BRS transformation laws (70) and (71) have to be added.

In this way we can associate a set of contact diagrams to every FD. Conversely, replacing a ghost line by a gauge boson line and the BRS transformed field by an external particle, we can associate exactly one FD to each contact diagram.

Since (in a linear R_ξ gauge) to every vertex of the ghosts corresponds a vertex of the gauge bosons, the contact terms generated in that way from the complete set of FDs must indeed be all the contact terms required by the STI. Therefore it is sensible to define

Definition 2 A subset of diagrams satisfies a STI if the contact terms obtained by the mapping \mathcal{F} agree with the result of contracting an external gauge boson.

and

Definition 3 A GIC is a subset of FDs that satisfies the STIs and in addition becomes independent of the gauge parameter when all external particles are on-shell.

4.2 Definition of gauge flips

As we will see below, we have to define the elementary gauge flips as the minimal set of four point diagrams with a given set of external particles and at least one external gauge boson, satisfying the STI. In a generic graphical notation, the gauge flips are denoted

8

$$\mathcal{G}_4 = \left\{ \begin{array}{c} \text{diagram 1} \\ \text{diagram 2} \\ \text{diagram 3} \\ \text{diagram 4} \end{array} \right\} \quad (26)$$

The internal particles appearing in these diagrams are determined by the requirement that the flips are the *minimal* set of diagrams satisfying the STIs. Of course, only diagrams allowed by the Feynman rules have to be included in each application of (26).

In R_ξ gauge also the corresponding four point functions with some or all external gauge bosons replaced by GBs appear as subamplitudes in larger diagrams. It may happen, that the minimal GIC for the gauge boson subamplitude does not coincide with the minimal GIC for the GB subamplitude. In this case, the gauge flips have to be defined in such a way that not only the gauge boson amplitudes, but also all corresponding GB amplitudes satisfy the STIs.

In the presence of quartic Higgs vertices, we will also need elementary flips among five point functions:

$$\mathcal{G}_5 = \left\{ \begin{array}{c} \text{diagram 1} \\ \text{diagram 2} \\ \text{diagram 3} \\ \text{diagram 4} \end{array} \right\} \quad (27)$$

4.3 Gauge parameter independence

We will now show on tree level that the gauge parameter independence of physical amplitudes is a consequence of the WIs of the theory. To obtain gauge parameter independent amplitudes, the ξ dependence of the propagators must cancel among the gauge boson and the GB exchange diagrams. To see how this works, we note that the gauge boson propagator in R_ξ gauge can be written as the propagator in unitarity gauge plus a term proportional to the GB propagator:

$$\begin{aligned} iD_W^{\mu\nu}(q) &= \frac{1}{q^2 - m_W^2} \left(g^{\mu\nu} - \frac{q^\mu q^\nu}{m_W^2} \right) + \frac{q^\mu q^\nu}{m_W^2} \frac{1}{q^2 - \xi m_W^2} \\ &= iD_{W,U}^{\mu\nu} - \frac{q^\mu q^\nu}{m_W^2} (iD_\phi). \end{aligned} \quad (28)$$

If we consider a gauge boson that is exchanged between two subamplitudes together with the corresponding GB, we see that the gauge parameter dependence cancels between the unphysical part of the gauge boson propagator and the GB propagator if the subamplitudes satisfy the WI (1). However, in general we *cannot* decompose a scattering matrix element into a sum over subamplitudes,

connected by *one* propagator:

$$\text{Diagram with } N \text{ external lines} \neq \sum_{i+j=N+2} \text{Diagram with } i \text{ external lines} - \text{Diagram with } j \text{ external lines} \quad (29)$$

For example, the grove G_s from (3) cannot be factorized into subamplitudes:

$$G_s \neq \text{Diagram 1} + \text{Diagram 2} \quad (30)$$

because the diagram

$$\text{Diagram 3} \quad (31)$$

contributes to both subamplitudes and would be counted twice.

Nevertheless, this problem can be avoided if we consider an infinitesimal change in the gauge parameter

$$\xi = \xi_0 + \delta\xi$$

and work to first order in $\delta\xi$. As usual, finite changes of ξ are generated by successive infinitesimal transformations.

Under an infinitesimal variation of ξ , the gauge boson propagator changes as (see (28))

$$D_{W,\xi}^{\mu\nu}(q) = D_{W,\xi_0}^{\mu\nu}(q) - \frac{iq^\mu q^\nu}{(q^2 - \xi_0 m_W^2)^2} \delta\xi + \mathcal{O}((\delta\xi)^2). \quad (32)$$

We will represent this decomposition graphically as

$$\text{Wavy line} = \text{Coiled line} + \text{Dotted line} \quad (33)$$

$$D_{W,\xi}^{\mu\nu}(q) = D_{W,\xi_0}^{\mu\nu}(q) - q^\mu q^\nu D_{c,\xi_0}^2(q) \delta\xi$$

Similarly the GB propagator becomes

$$D_{\phi\xi} = D_{\phi\xi_0} + m_W^2 \frac{i}{(q^2 - \xi_0 m_W^2)^2} \delta\xi + \mathcal{O}((\delta\xi)^2). \quad (34)$$

Inserting the decomposition (33) into the diagram (31) we get two contributions linear in $\delta\xi$. Therefore we can factorize the contributions linear in $\delta\xi$:

$$\partial_\xi G_s = \text{Diagram 4} + \text{Diagram 5} \quad (35)$$

Using the WIs for the subamplitudes, we see that the gauge parameter dependence of the grove G_s vanishes.

For general amplitudes, the terms linear in $\delta\xi$ can be factorized in a similar way. To see this, we regard the unphysical propagators as new “particles” with the appropriate Feynman rules. The $\mathcal{O}(\delta\xi)$ contribution to the GF consists of FDs where the new particle appears *exactly* once and no double counting occurs. The same reasoning can be applied to the variation of the GB propagator.

Therefore, the parts of the propagators linear in $\delta\xi$ connect complete subamplitudes that satisfy the WIs and as a result the amplitude is gauge parameter independent:

$$\partial_\xi \text{Diagram with } \xi \text{ external lines} = \sum_{i+j=N+2} \text{Diagram with } i \text{ external lines} - \text{Diagram with } j \text{ external lines} = 0. \quad (36)$$

If the external particles are off-shell, we have to use the STIs instead of the WIs and the GFs become ξ dependent.

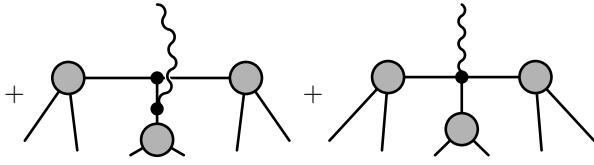
4.4 Gauge invariance classes

We are now ready to show that the groves obtained by the gauge flips defined as in Sect. 4.2 are indeed the minimal GICs according to definition 3. For amplitudes without external gauge bosons, GICs are defined as subsets of FDs that are gauge parameter independent when all external particles are on their mass shell.

From (36) we know that it suffices that the parts of the propagators linear in $\delta\xi$ connect subamplitudes satisfying the WIs in order to obtain gauge parameter independent quantities. As induction hypothesis, we assume that it has been shown that the $N - 1$ particle diagrams connected by gauge flips satisfy the STIs and are gauge parameter independent. Therefore applying gauge flips to all *internal* gauge bosons of a N -point function ensures its gauge parameter independence. Thus the case of amplitudes without external gauge bosons is reduced to the discussion of amplitudes with fewer external particles and external gauge bosons.

Now we consider the insertion of a gauge boson into a FD with $N - 1$ external particles. We pick out an arbitrary cubic vertex of the diagram and insert the gauge boson into all three legs of the vertex and include a quartic vertex (if allowed by the Feynman rules):

$$\text{Diagram 6} \equiv \text{Diagram 7} + \text{Diagram 8} \quad (37)$$



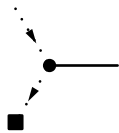
These diagrams are connected by the gauge flips \mathcal{G}_4 of (26). The insertion of a contracted gauge boson into a FD via a three point vertex can be evaluated using the STI (21) and the identity (19):

(38)

For internal gauge bosons the additional pieces (22) in the STIs contribute terms of the form

(39)

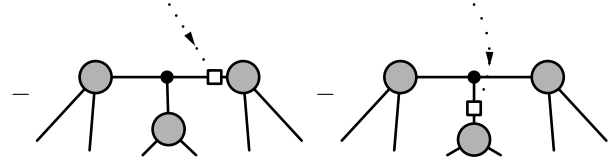
and similarly for GBs. Here the subdiagram



has to be understood as the insertion of a contracted ghost vertex, no internal ghost propagator appears. Note that the diagrams of this form only appear for particles that couple simultaneously to two gauge bosons and to ghosts.

For the remaining diagrams in (37) involving cubic vertices, we can repeat the same manipulations and find, using the STI of the four point vertex (23), that everything cancels apart from the terms

(40)



The contributions (39) exclusive to internal gauge bosons and GBs will be discussed below.

To cancel the remaining diagrams in (40), we have to “zoom in” into the blobs, insert the external gauge boson at the next vertex and repeat the same procedure for the next vertices. This will cancel the terms from (40) but leaves new terms of the same form at the next vertices. This process can be iterated until the external particles are reached. The remaining terms are the contact terms of the STI for the GF with the ghost line going through the diagram without interaction:

(41)

Clearly, the procedure discussed above amounts to gauge-flipping the external gauge boson through the original diagram. The contact terms generated this way are among those generated by the mapping \mathcal{F} (25) from diagrams with an insertion of the gauge boson into an external leg.

The contraction of a gauge boson inserted adjacent to a quartic Higgs vertex can be treated analogously. This time the cancellation takes place because of the STI (24). It involves the diagrams connected by the five point flips (27):

(42)

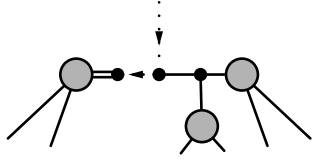
Again, these terms can be canceled by iterating this procedure at the next vertices in the blobs until the external particles are reached. In a non-linear realization of the symmetry, the cancellation of the contributions of higher vertices involving Goldstone bosons proceeds in the same way.

The cancellation mechanism just described involves only the STIs of the vertices where the gauge boson is inserted and thus will be called “first order cancellations”. As we have seen, the first order cancellations do not require the introduction of diagrams of a different topology.

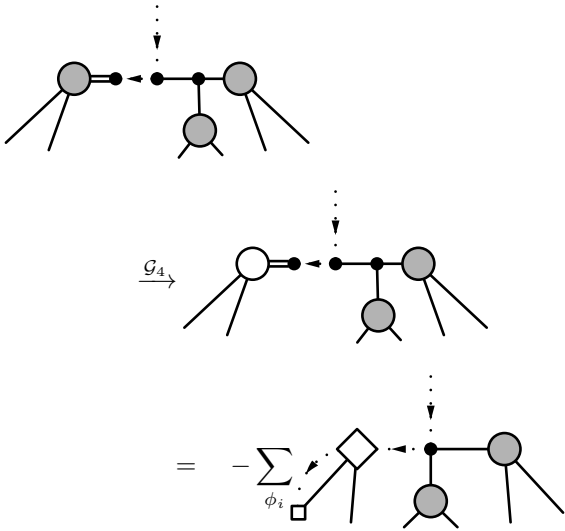
Let us now turn to the cancellation of the terms of the form (39). They will lead to the prescription to flip also all internal gauge bosons and will be called “second order cancellations” because they involve not only the STI for the vertex with the gauge boson insertion but a STI for

another subamplitude. In contrast to the first order ones, they force us to introduce diagrams of a different topology.

Combining the GB and gauge boson diagrams, the diagrams in question have the form

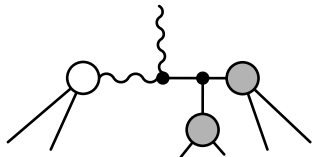

(43)

To proceed, we have to add additional diagrams so that we can use a STI for the subamplitude connected to the double line. We will proceed by induction and assume that the $N - 1$ particle groves satisfy the STI (73) in the sense of definition 2. After applying the gauge flips to the subamplitude connected to the double line, we can use the STI and obtain


(44)

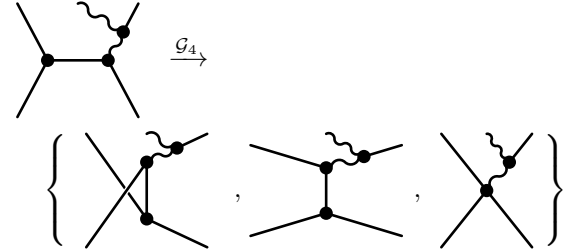
Of course now we have to flip the external gauge boson also through the new diagrams.

Since by assumption the contact terms of the subamplitude are those generated by the formal mapping \mathcal{F} , we see that the contact terms are those corresponding to the diagrams

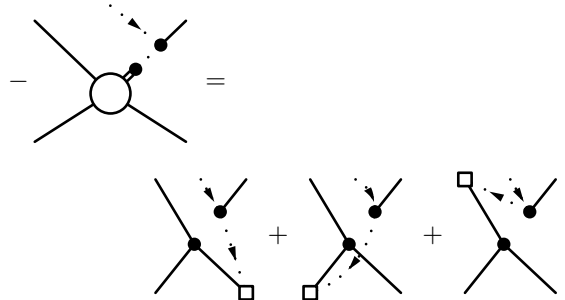

(45)

This shows that for the set of diagrams obtained by applying the gauge flips, the contact terms that appear by contracting an external gauge boson are indeed the same ones as are assigned to every diagram by the mapping \mathcal{F} and therefore the STI is satisfied in the sense of definition 2. The sets of diagrams connected by gauge flips are indeed the *minimal* GICs since, by construction, an omission of a diagram would lead to an violation of a WI. This concludes our proof.

As an example, we discuss five point functions in a general SGBT. We have seen in Sect. 2.1 that the five point function with four external matter fields can be decomposed into groves like (3). Such a decomposition is no longer possible if the five point amplitude contains more than one gauge external boson or external Higgs bosons (in a linear representation of the symmetry). This can be understood using the techniques of our proof: if the external gauge boson is inserted into an external Higgs or gauge boson leg, it can couple to an internal gauge boson so we have to apply the gauge flips to obtain


(46)

This brings in t - and u -channel diagrams and the quartic vertices. Adding these diagrams and the corresponding GB diagrams, we can use the STI for the four point function and a “second order cancellation” can take place:


(47)

These are some of the contact terms obtained by applying the mapping (25) to the diagrams of (46). Of course now we have to flip the external gauge boson through all four diagrams in (46) and this will in general result in *all* diagrams of the amplitude.

We can also see the reason for the groves appearing in the $\bar{q}q \rightarrow \bar{q}qg$ amplitude (3). Because of fermion number conservation, diagrams of the form of (46) cannot be generated by the insertion of a gauge boson into a fermion line. Since these are the only diagrams that force us to perform the internal gauge flips that bring in the t - and u -channel diagrams, we see again that the grove (3) is indeed a GIC for external fermions.

5 Definition of gauge flips in spontaneously broken gauge theories

5.1 Gauge flips for linearly realized symmetries

In order to apply the formalism of flips and groves to SGBTs, we have to define the elementary flips of the theory. According to [6] and the results of Sect. 4, the gauge

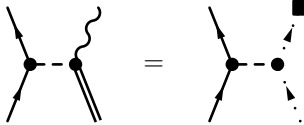
flips are given by the minimal sets of four point diagrams satisfying the STIs. We first treat the case of the amplitude $\bar{f}f \rightarrow WW$ in some detail, before extending the discussion to the remaining elementary flips.

To see the origin of the subtleties in the definition of the gauge flips including Higgs propagators, we point out a feature of the conditions arising from the WIs of SGBTs. The evaluation of the WI (1) for the process $\bar{f}f \rightarrow WW$ in a general SGBT [13] leads to the same condition as in an unbroken gauge theory, i.e. the Lie algebra structure of the fermion–gauge boson couplings. No such conditions arise for the Higgs couplings because the Higgs exchange diagram $G_{4,2F}^4$ from (7) satisfies the WI by itself. The reason for this can be seen from the STI for the HWW vertex, resulting from the STI (20) with the additional ghost term (22a):

$$\begin{aligned} & -\langle \mathcal{D}_a(p_a) W_b^\mu(p_b) H_i(p_i) \rangle^{1\text{PI}} \\ &= i \frac{1}{\xi} p_b^\mu \langle c_a(p_a) \bar{c}_b(p_b) H_i(p_i) \rangle^{1\text{PI}}. \end{aligned} \quad (48)$$

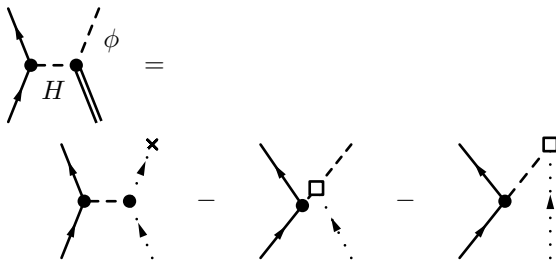
Here we have used that W –Higgs and W –GB mixing vanishes at tree level. Contracting with a physical polarization vector of the gauge boson, the ghost term drops out and we find that a simple WI (1) for the HWW vertex is valid even if the Higgs is off-shell.

For external particles off their mass shell, the STI (48) implies that the Higgs diagram in the $\bar{f}fWW$ amplitude reproduces a ghost diagram from the STI:


(49)

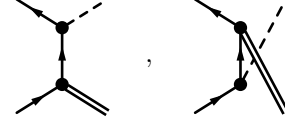
According to the definitions in Sect. 4.1, this means that this diagram satisfies the STI by itself. Therefore we would conclude that the elementary gauge flips in a SGBT are still defined by (6b). However, the recursive proof from Sect. 4, requires that the set of GB diagrams corresponding to the elementary gauge flips also satisfies the STIs by themselves. As we will now show, this forces us to include the Higgs exchange diagram in the gauge flips.

Considering the Higgs exchange diagram in the $\bar{f}f \rightarrow W\phi$ amplitude, we see, using the STI for the $WH\phi$ vertex, that there are additional contributions in this case:


(50)

The first and the last diagram are contact terms required by the STI. To cancel the second diagram, we are forced

to add two more diagrams:



According to the general analysis in Sect. 4, these terms provide the remaining diagrams, so a cancellation because of the STI


(51)

takes place. Therefore, our definition in Sect. 4.2 forces us to include the diagrams $\tilde{G}_{4,2F}^1$ and $\tilde{G}_{4,2F}^2$ in the gauge flips for $\bar{f}f \rightarrow WW$. But then also the diagram $\tilde{G}_{4,2F}^3$ with a triple gauge boson vertex has to be included and we obtain the correct set of flips given in (7).

This discussion can easily be generalized to the other elementary gauge flips since the argument did depend only on the structure of the STIs for the WWH and $W\phi H$ vertices. Considering the remaining vertices, we only used the fact that they satisfy the appropriate STIs. Therefore the conclusions apply also for the other elementary flips and we have to include the Higgs exchange diagrams in *all* gauge flips. All flips including new elementary gauge flips for gauge–Higgs boson four point functions are displayed in Appendix B.

5.2 Flips for non-linear realizations of symmetries

It is not obvious that the intuitive arguments of Sect. 2.2 for the simplifications of the gauge flips in a non-linear realization of the symmetry carry over to theories with a more complicated Higgs sector where the Higgs bosons transform non-trivially under the unbroken subgroup. To demonstrate that this is indeed the case, we will show for a general non-linearly realized symmetry that the STI for the $WH\phi$ vertex becomes trivial at tree level. Therefore, according to the discussion in the previous subsection, the Higgs exchange diagrams have not to be included in the gauge flips without external Higgs bosons.

We consider a symmetry group G that is spontaneously broken down to a subgroup $H \subset G$. The GBs can be used to parameterize the coset space G/H by introducing the exponential representation

$$U(\phi) = e^{\frac{i}{f} \phi_a V^a} \in G/H, \quad (52)$$

where the V^a are the broken generators. The generators of the unbroken subgroup will be called L^a .

To derive the STIs, we need the BRS transformations of fields in the non-linear representation. They can be obtained in the usual way from the infinitesimal gauge transformations by replacing the gauge parameter ω by a ghost field. Under the gauge symmetry, the GBs transform non-linearly:

$$\phi_a \rightarrow \phi'_a(\phi, \omega) \quad (53)$$

On fermions, other matter fields and Higgs bosons, gauge transformations of the full gauge group are realized as linear, ϕ dependent transformations of the unbroken subgroup:

$$\Phi' = \mathcal{H}(\phi, \omega)\Phi. \quad (54)$$

The explicit form of the functions ϕ' and \mathcal{H} can be found in the literature [7].

For the derivation of the BRS transformations in theories with non-linear symmetries, we consider the transformations (53) and (54) for the infinitesimal parameters $\omega = \epsilon$. To linear order in ϵ , we can write

$$\begin{aligned} U'(\phi, \epsilon) &= U(\phi'(\phi, \epsilon)) = 1 + i\mathcal{K}_a^b(\phi)\epsilon_b V^a + \mathcal{O}(\epsilon^2) \\ \mathcal{H}(\phi, \epsilon) &= 1 + i\Omega_a^b(\phi)\epsilon_b L^a + \mathcal{O}(\epsilon^2). \end{aligned} \quad (55)$$

We can now introduce the BRS transformations

$$\begin{aligned} \delta_{\text{BRS}}\phi_a &= f_{cb}\mathcal{K}_a^b(\phi) \\ \delta_{\text{BRS}}\Phi &= ic_a\Omega^a(\phi)\Phi, \end{aligned} \quad (56)$$

with $\Omega^a = \Omega_b^a L^b$. Below, we will only need the first order in ϕ :

$$\begin{aligned} \mathcal{K}_a^b(\phi) &= g_a^b + t_{ac}^b\phi_c + \mathcal{O}(\phi^2) \\ \Omega_{ij}^a(\phi) &= \mathcal{T}_{ij}^a + \mathcal{O}(\phi). \end{aligned} \quad (57)$$

We will still use a linear R_ξ gauge fixing instead of a gauge fixing function in terms of the U . Note that the gauge fixing term in the non-linear parameterization contains no Higgs-ghost interaction since the Higgs bosons do not appear in the BRS transformation of the GBs.

Turning to the STI for the $WH\phi$ vertex that is relevant for the discussion of gauge flips, after setting classical fields and sources to zero, the terms of higher order in ϕ drop out and we arrive at:

$$\begin{aligned} & -\langle \mathcal{D}_a(p_a)\phi_b(p_b)H_i(k_i) \rangle^{\text{1PI}} \\ &= f t_{cb}^a \langle \phi_c(p_a + p_b)H_i(k_i) \rangle^{\text{1PI}} \\ &+ i\mathcal{T}_{ji}^a \langle \phi_b(p_b)H_j(-p_b) \rangle^{\text{1PI}} \stackrel{\text{tree level}}{=} 0. \end{aligned} \quad (58)$$

On tree level, the right hand side vanishes because there is no Higgs-GB mixing. Thus we can conclude that the diagram



$$(59)$$

satisfies the WI by itself and therefore the corresponding gauge boson diagram $G_{4,2F}^4$ has not to be included in the gauge flips. In higher orders of perturbation theory, the right hand side of (58) no longer vanishes since a ϕ - H mixing is generated by loop diagrams.

No simplification compared to the linear case appears for the STI for the HHW vertex

$$\begin{aligned} & -\langle \mathcal{D}_a(p_a)H_i(k_i)H_j(k_j) \rangle^{\text{1PI}} \\ &= i\mathcal{T}_{ki}^a \langle H_k(-p_j)H_j(p_j) \rangle^{\text{1PI}} + i\mathcal{T}_{kj}^a \langle H_i(p_i)H_k(-p_i) \rangle^{\text{1PI}} \end{aligned} \quad (60)$$

This identity is similar to the linear case [13] and, as has already been stated in Sect. 2.2, Higgs exchange diagrams with a HHW vertex have to be included in the gauge flips also in the non-linear realization.

In contrast to a linear realization of the symmetry, derivatives of the BRS transforms with respect to more than one field contribute to the STIs for vertices with more than three external particles involving GBs. For vertices with one GB, the STI (17) reads

$$\begin{aligned} 0 &= \sum_{\Psi} \int d^4x \{ \Gamma_{c_a\Psi^*} \Gamma_{\Psi\Phi_1\Phi_2\phi} \\ &+ [\Gamma_{c_a\Psi^*\Phi_1} \Gamma_{\Psi\Phi_2\phi} + \Gamma_{c_a\Psi^*\Phi_1\phi} \Gamma_{\Psi\Phi_2} + (1 \leftrightarrow 2)] \}. \end{aligned} \quad (61)$$

In the STI for the $W\phi HH$ vertex, we get a contribution from the higher derivatives of the form

$$\Gamma_{\bar{c}_a H_k^*} H_j \phi_b \Gamma_{H_k H_i}$$

while the similar contributions to the STI for the $WW\phi H$ vertex involve mixed two point functions that vanish at tree level. Therefore diagrams like



$$(62)$$

do not force us to include additional flips involving the internal Higgs bosons. The key feature of the non-linear realization ensuring this simplification is that there is no term $\propto c\phi$ in the BRS-transform of the Higgs field.

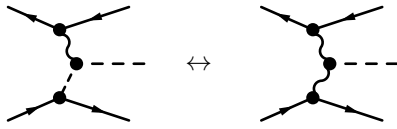
6 Groves in spontaneously broken gauge theories

6.1 Groves for linearly realized symmetries

To analyze the groves in SGBTs, we have implemented the flips described in Sect. 2.2 in the program bocages [8]. As an example for the structure of the groves, we consider the amplitude for the process $\bar{f}f \rightarrow \bar{f}fH$. The only new features compared to the QCD example $\bar{q}q \rightarrow \bar{q}qg$ from (3) are single diagram groves consisting of diagrams without gauge bosons. The remaining groves are similar to the groves in QCD with the external gluon replaced by a Higgs boson.

Similarly, in the process $\bar{f}f \rightarrow \bar{f}fW$ there appear only two groves, like in the case of QCD discussed in Sect. 2.1. They include, of course, additional diagrams involving Higgs bosons.

Apart from the one diagram groves, the Higgs flips (79) do not lead to additional groves. If there is at least one gauge boson in a diagram, the gauge flips can always be used to arrive at diagrams with more than one internal gauge boson. For example the diagrams



are connected by a gauge flip from (8).

It turns out that this is the generic structure: the only new groves compared to the case of unbroken gauge theories consist of one diagram each, where all internal particles are Higgs bosons. The remaining diagrams fall in the same groves that have been discussed in Sect. 2.1.

Since the Higgs–fermion Yukawa couplings are proportional to the fermion masses, the couplings of Higgs bosons to light fermions are usually set to zero in practical calculations. Of course this is only a consistent approximation if the masses of light fermions are set to zero at the same time. The set of diagrams obtained by neglecting the coupling to light fermions in general does *not* correspond to a gauge invariant subset if the fermion masses are not set to zero. In practice, the numerical instabilities caused by this inconsistency are negligible but there are small corners in phase space where they can become relevant.

6.2 Groves for non-linearly realized symmetries

In the case of non-linearly realized symmetries, the gauge flips simplify as we have discussed in Sect. 2.2, so there is a more interesting structure of the groves than in the case of linearly parameterized scalar sectors.

An especially interesting structure of the groves appears in theories with a single, neutral Higgs boson and a non-linearly realized symmetry. This corresponds to a non-linear realization of the minimal electroweak SM [11]. In this parameterization, the Higgs boson is not connected to the symmetry breaking mechanism but merely an additional matter particle. Even though the Higgs is no longer an essential ingredient of the theory, the inclusion of the Higgs boson in a non-linearly realized theory is useful to study anomalous Higgs–gauge boson couplings [14]. For a Higgs boson that is a singlet under the unbroken symmetry group, there is no HHW vertex. We see from the gauge flips in (76), that in this case the gauge flips “conserve” Higgs number in the sense that only external Higgs bosons appear. Therefore gauge flips cannot change the number of Higgs bosons and the groves can be classified according to the number of internal Higgs bosons.

In theories with a general scalar sector, this “Higgs conservation law” breaks down but the number of groves is still larger than in the case of a linearly realized symmetry.

The reduced number of gauge flips has also consequences for unitarity gauge. Unitarity gauge can equivalently be defined as the limit $\xi \rightarrow \infty$ of the linearly realized theory in R_ξ gauge or from non-linearly realized symmetries by transforming the GBs away. Therefore it follows that the groves obtained from the reduced sets of flips are also consistent in unitarity gauge, i.e. they satisfy the appropriate WIs. Although it is not sensible to speak of “gauge invariance classes” in a fixed gauge, this result nevertheless indicates that no numerical instabilities due to violations of WIs will appear.

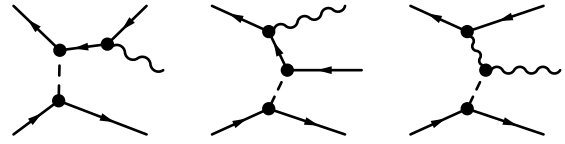
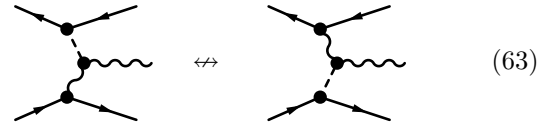
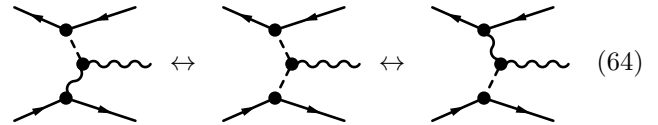


Fig. 2. Mixed grove for $\bar{f}f \rightarrow \bar{f}fW$ for a singlet Higgs

As example, we consider the process $\bar{f}f \rightarrow \bar{f}fW$. Let us discuss the case of a single, neutral Higgs boson first. We find that the amplitude for $\bar{f}f \rightarrow \bar{f}fW$ can be decomposed into six groves instead of the two appearing in unbroken gauge theories and in the linear parameterization of SGBTs. Two groves are “gauge groves” consisting of five diagrams without internal Higgs boson that look exactly like in QCD (3). The remaining groves are “mixed groves” with one internal Higgs boson. An example of a mixed grove is given in Fig. 2. Here all diagrams are proportional to the coupling of the Higgs to one fermion pair, but this is not always the case as we will see below. In this example, two mixed groves correspond to one gauge grove. This additional structure arises because there is no gauge flip between the two diagrams



In a non-linear realization of a more complicated Higgs sector, the Higgs bosons transform according to a linear representation of the unbroken subgroup and therefore may couple to the massless gauge boson through HHW vertices. This enables an indirect flip between the two diagrams from (63) since the flips (8) have to be included in the gauge flips also for the non-linear parameterization



We see that the “Higgs conservation” in the gauge flips breaks down. Therefore the appearance of Higgs bosons charged under the unbroken subgroup reduces the number of groves. One finds that mixed groves are still present, however, in general only *one* mixed grove corresponds to a gauge grove and they do not contain a fixed number of Higgs bosons. The same structure is found in amplitudes with more external particles. Let us first consider the six fermion amplitude. For a single Higgs boson, 18 mixed groves are obtained by inserting a fermion–antifermion pair via a Higgs boson in all possible places in the gauge groves of the four fermion amplitude. An application to the process $e^+e^- \rightarrow b\bar{b}t\bar{t}$ that is relevant for the measurement of the top Yukawa coupling is shown in Fig. 3. Again all the diagrams in the mixed groves are proportional to the coupling of the Higgs to one fermion pair. For a general Higgs sector, because of the additional flips as in (64) only six mixed groves remain, corresponding to the gauge groves.

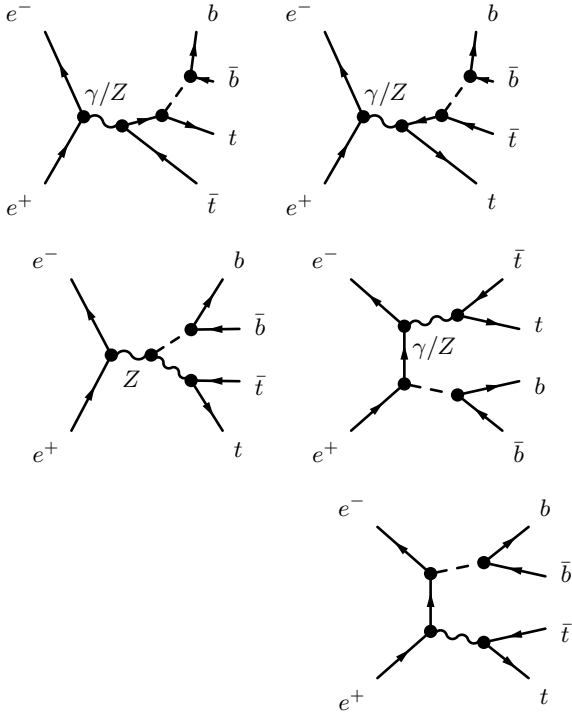


Fig. 3. Mixed grove in the $e^+e^- \rightarrow b\bar{b}t\bar{t}$ amplitude

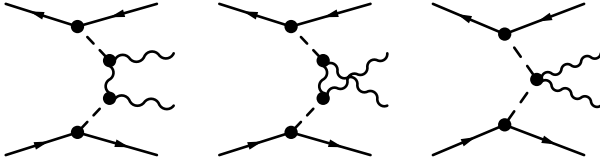
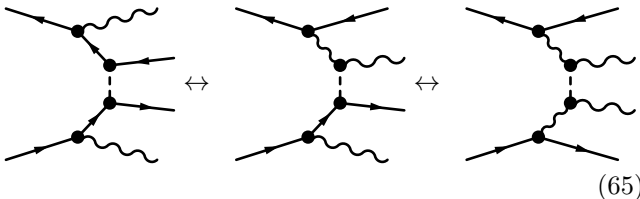


Fig. 4. Mixed grove with two Higgs bosons in the 4 fermion 2 gauge boson amplitude

As a final example we discuss the amplitude $\bar{f}f \rightarrow \bar{f}f WW$. For a single Higgs boson, the Higgs number conservation leads to the appearance of two mixed groves with two Higgs bosons. One example is shown in Fig. 4.

Furthermore, apart from two gauge groves (and several one diagram groves), there are ten mixed groves with one Higgs boson. In general it is not true that all mixed groves are proportional to one Higgs coupling. An example is provided by the three diagrams



that are connected by flips from (8) and that have no Higgs coupling in common. Again, in a more general Higgs sector with HHW vertices, the “Higgs conservation law” breaks down and only two mixed groves remain.

We have checked the results of this section numerically for processes with up to eight external particles in

the SM [13], using the optimizing matrix element generator O’Mega [15]. We have considered the “gauge groves” that can be obtained by setting all Higgs couplings to zero. As expected, this is consistent in unitarity gauge, i.e. all WIs are satisfied. In R_ξ gauge, the WIs for GFs with four external particles are satisfied while the WIs with five external particles are violated badly. This has to be expected, since according to the discussion in Sect. 5 the four particle gauge boson exchange diagrams satisfy the WI, but the four point diagrams with external GBs that appear in the five point functions violate the WIs. Starting from the six point functions, these violations of the five point WIs cause inconsistencies between the matrix elements in unitarity gauge and R_ξ gauge.

7 Summary and outlook

We have given a new proof for the formalism of flips [6] for the determination of gauge invariant subsets of FDs (groves). Our proof clarifies the precise definition of gauge flips in SGBTs that has been applied to the classification of the GICs. We found new GICs in theories with a nonlinearly realized scalar sector. In this case the groves in theories with only neutral Higgs bosons can be classified according to the number of internal Higgs boson lines. These results are also relevant for calculations in unitarity gauge. In theories with a linearly realized scalar sector in R_ξ gauge, no additional non-trivial groves compared to the unbroken case exist. The applications of gauge flips to loop diagrams is currently being studied [16] and the extension of our proof to loop diagrams, using the Feynman tree theorem [17] is under investigation. Our approach might also be useful for the extensions of groves to supersymmetric theories, using the results of [18].

Acknowledgements. We thank D. Ondreka for useful discussions. This work has been supported by the Bundesministerium für Bildung und Forschung Germany (05HT1RDA/6).

A STIs for Green’s functions

In this appendix we set up a graphical notation for BRS transformations and STIs for GFs. We work in a general quantum field theory, denoting the physical fields and GBs collectively by Φ :

$$\Phi = \{\psi, W^\mu, H, \phi, \dots\}.$$

All fields including ghosts and auxiliary fields will be denoted by Ψ :

$$\Psi = \{\Phi, c, \bar{c}, B\}.$$

The STI for GFs reads

$$\begin{aligned} &\langle \text{out} | T[B \Phi_1 \dots \Phi_n] | \text{in} \rangle \\ &= \sum_i (\pm) \langle \text{out} | T[\bar{c} \Phi_i \dots \delta_{\text{BRS}} \Phi_i \dots \Phi_n(y_n)] | \text{in} \rangle, \end{aligned} \quad (66)$$

(76a)

For non-linear realizations, the Higgs exchange diagrams \tilde{G}_4^5 , \tilde{G}_4^6 and \tilde{G}_4^7 are not present in the gauge flips.

The flips for $\bar{f}f \rightarrow WW$ are in the linear representation

$$\tilde{G}_{4,2F} = \{\tilde{G}_{4,2F}^i | i = 1, 2, 3, 4\} =$$

(76b)

Again, the Higgs exchange diagram $\tilde{G}_{4,2F}^4$ is not present for non-linear symmetries.

The gauge $\bar{f}f \rightarrow WH$ flips are for linear and non-linear realizations

$$\tilde{G}_{4,1H2F} = \{\tilde{G}_{4,1H2F}^i | i = 1, 2, 3, 4\} =$$

(76c)

A new feature in SBGTs are the $3W-1H$ flips that have the same form in linear and non-linear realizations:

$$\tilde{G}_{4,1H} = \{\tilde{G}_{4,1H}^i | i = 1, \dots, 6\} =$$

(76d)

The $2W2H$ flips are for linear symmetries:

$$\tilde{G}_{4,2H} = \{\tilde{G}_{4,2H}^i | i = 1, \dots, 7\} =$$

(76e)

Here the diagram $\tilde{G}_{4,2H}^6$ is not included in the gauge flips for non-linear realizations of the symmetry.

Finally we have the $3HW$ flips that again have the same form in linear and non-linear realizations:

$$\tilde{G}_{4,3H} = \{\tilde{G}_{4,3H}^i | i = 1, \dots, 6\} =$$

(77)

B.2 Flavor and Higgs flips

Higgs exchange has to be included in the flavor flips so they are given by

$$\tilde{F}_4 = \{\tilde{F}_4^i | i = 1, \dots, 6\} =$$

(78)

Finally there are ‘Higgs flips’ for diagrams without external gauge bosons that are gauge parameter independent by themselves:

$$\tilde{H}_4 = \{\tilde{H}_4^i | i = 1, \dots, 7\} =$$

(79a)

and

$$\tilde{H}_{4,2F} = \{\tilde{H}_{4,2F}^i | i = 1, 2, 3, 4\} =$$

(79b)

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